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L. D'Amico

- | Descriptive Statistics |       | Descriptive Statistics |                     | Descriptive Statistics |      |
|------------------------|-------|------------------------|---------------------|------------------------|------|
| Variable               | Mean  | SD                     | Variable            | Mean                   | SD   |
| Age                    | 35.2  | 12.5                   | Gender              | Male                   | 45.5 |
| Education              | 12.8  | 2.1                    | Marital Status      | Married                | 52.3 |
| Income                 | 25.4  | 8.7                    | Occupation          | Professional           | 38.9 |
| Health Status          | 78.5  | 15.2                   | Religion            | Christian              | 65.1 |
| Stress Level           | 65.3  | 22.1                   | Life Satisfaction   | High                   | 42.7 |
| Depression Score       | 15.6  | 10.4                   | Resilience Score    | 55.8                   | 18.3 |
| Life Satisfaction      | 55.2  | 14.5                   | Optimism Score      | 62.4                   | 16.7 |
| Resilience Score       | 58.7  | 19.2                   | Self-Efficacy Score | 68.9                   | 17.5 |
| Optimism Score         | 65.1  | 15.8                   | Gratitude Score     | 72.3                   | 14.2 |
| Self-Efficacy Score    | 70.5  | 16.3                   | Forgiveness Score   | 78.6                   | 13.9 |
| Gratitude Score        | 75.8  | 15.1                   | Compassion Score    | 82.1                   | 12.7 |
| Forgiveness Score      | 80.2  | 14.5                   | Kindness Score      | 85.4                   | 11.8 |
| Kindness Score         | 88.7  | 13.2                   | Generosity Score    | 90.1                   | 12.5 |
| Generosity Score       | 92.5  | 11.9                   | Patience Score      | 95.3                   | 10.7 |
| Patience Score         | 97.8  | 10.2                   | Humility Score      | 99.2                   | 9.5  |
| Humility Score         | 100.0 | 8.8                    | Modesty Score       | 100.0                  | 8.1  |
| Modesty Score          | 100.0 | 7.5                    | Meekness Score      | 100.0                  | 7.2  |
| Meekness Score         | 100.0 | 6.8                    | Lowliness Score     | 100.0                  | 6.5  |
| Lowliness Score        | 100.0 | 6.1                    | Softness Score      | 100.0                  | 5.9  |
| Softness Score         | 100.0 | 5.6                    | Gentleness Score    | 100.0                  | 5.3  |
| Gentleness Score       | 100.0 | 5.0                    | Peaceableness Score | 100.0                  | 4.8  |
| Peaceableness Score    | 100.0 | 4.5                    | Goodwill Score      | 100.0                  | 4.2  |
| Goodwill Score         | 100.0 | 4.0                    | Kindness Score      | 100.0                  | 3.8  |
| Kindness Score         | 100.0 | 3.5                    | Generosity Score    | 100.0                  | 3.2  |
| Generosity Score       | 100.0 | 3.0                    | Patience Score      | 100.0                  | 2.8  |
| Patience Score         | 100.0 | 2.5                    | Humility Score      | 100.0                  | 2.2  |
| Humility Score         | 100.0 | 2.0                    | Modesty Score       | 100.0                  | 1.8  |
| Modesty Score          | 100.0 | 1.5                    | Meekness Score      | 100.0                  | 1.2  |
| Meekness Score         | 100.0 | 1.0                    | Lowliness Score     | 100.0                  | 0.8  |
| Lowliness Score        | 100.0 | 0.5                    | Softness Score      | 100.0                  | 0.3  |
| Softness Score         | 100.0 | 0.2                    | Gentleness Score    | 100.0                  | 0.1  |
| Gentleness Score       | 100.0 | 0.0                    | Peaceableness Score | 100.0                  | 0.0  |
| Peaceableness Score    | 100.0 | 0.0                    | Goodwill Score      | 100.0                  | 0.0  |
| Goodwill Score         | 100.0 | 0.0                    | Kindness Score      | 100.0                  | 0.0  |
| Kindness Score         | 100.0 | 0.0                    | Generosity Score    | 100.0                  | 0.0  |
| Generosity Score       | 100.0 | 0.0                    | Patience Score      | 100.0                  | 0.0  |
| Patience Score         | 100.0 | 0.0                    | Humility Score      | 100.0                  | 0.0  |
| Humility Score         | 100.0 | 0.0                    | Modesty Score       | 100.0                  | 0.0  |
| Modesty Score          | 100.0 | 0.0                    | Meekness Score      | 100.0                  | 0.0  |
| Meekness Score         | 100.0 | 0.0                    | Lowliness Score     | 100.0                  | 0.0  |
| Lowliness Score        | 100.0 | 0.0                    | Softness Score      | 100.0                  | 0.0  |
| Softness Score         | 100.0 | 0.0                    | Gentleness Score    | 100.0                  | 0.0  |
| Gentleness Score       | 100.0 | 0.0                    | Peaceableness Score | 100.0                  | 0.0  |
| Peaceableness Score    | 100.0 | 0.0                    | Goodwill Score      | 100.0                  | 0.0  |
| Goodwill Score         | 100.0 | 0.0                    | Kindness Score      | 100.0                  | 0.0  |
| Kindness Score         | 100.0 | 0.0                    | Generosity Score    | 100.0                  | 0.0  |
| Generosity Score       | 100.0 | 0.0                    | Patience Score      | 100.0                  | 0.0  |
| Patience Score         | 100.0 | 0.0                    | Humility Score      | 100.0                  | 0.0  |
| Humility Score         | 100.0 | 0.0                    | Modesty Score       | 100.0                  | 0.0  |
| Modesty Score          | 100.0 | 0.0                    | Meekness Score      | 100.0                  | 0.0  |
| Meekness Score         | 100.0 | 0.0                    | Lowliness Score     | 100.0                  | 0.0  |
| Lowliness Score        | 100.0 | 0.0                    | Softness Score      | 100.0                  | 0.0  |
| Softness Score         | 100.0 | 0.0                    | Gentleness Score    | 100.0                  | 0.0  |
| Gentleness Score       | 100.0 | 0.0                    | Peaceableness Score | 100.0                  | 0.0  |
| Peaceableness Score    | 100.0 | 0.0                    | Goodwill Score      | 100.0                  | 0.0  |
| Goodwill Score         | 100.0 | 0.0                    | Kindness Score      | 100.0                  | 0.0  |
| Kindness Score         | 100.0 | 0.0                    | Generosity Score    | 100.0                  | 0.0  |
| Generosity Score       | 100.0 | 0.0                    | Patience Score      | 100.0                  | 0.0  |
| Patience Score         | 100.0 | 0.0                    | Humility Score      | 100.0                  | 0.0  |
| Humility Score         | 100.0 | 0.0                    | Modesty Score       | 100.0                  | 0.0  |
| Modesty Score          | 100.0 | 0.0                    | Meekness Score      | 100.0                  | 0.0  |
| Meekness Score         | 100.0 | 0                      |                     |                        |      |

a) Basic Fee					\$ 690.00
b) Independent Claims	<u>3</u>	- 3 =	<u>0</u>	x \$ 78.00 =	\$ <u>0</u>
c) Total Claims	<u>14</u>	- 20 =	<u>0</u>	x \$ 18.00 =	\$ <u>0</u>
d) Fee for Multiple Claims				\$260.00 =	\$ <u>          </u>

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1                    ITERATIVE MMSE EQUALIZATION-DECODER SOFT  
2                    INFORMATION EXCHANGE DECODING METHOD AND DEVICE

3                    FIELD OF THE INVENTION

4                    The present invention concerns the field of decoding of information symbols  
5                    received over a channel.

6                    BACKGROUND OF THE INVENTION

7                    Information symbols are transmitted over channels in most information, storage  
8                    and communication devices. The channel can comprise, for example, a wired or wireless  
9                    communication link between a transmitter or receiver, or an information path within a disk  
10                    drive. More generally, any information path of limited bandwidth through which digital  
11                    signals are transmitted may be defined as a channel. Such channels are subject to various  
12                    types of distortion, e.g., additive noise and intersymbol interference. These distortions limit  
13                    the data rate efficiencies for communication over such channels. Accordingly, much effort  
14                    has been devoted to reducing the effect of such distortions in the channels. The obvious end  
15                    goal is to decode symbols accurately. More specifically, the goal is to have low decoding  
16                    error rates. A second and conflicting goal is to increase the ratio of data symbols to total  
17                    channel symbols transmitted, i.e., the data rate.

18                    Recent focus for joint equalization and decoding has been on iterative decoding  
19                    schemes, and in particular, turbo coding schemes. These schemes are confidence building  
20                    schemes in which an equalizer and decoder trade soft information in the form of symbol  
21                    estimates until convergence is reached and hard decisions for symbols are output. These  
22                    iterative schemes treat the channel contribution like an error correction code. The data

1 symbols are generally permuted before transmission and are typically protected with a  
2 convolutional error correcting code. On the decoding side, soft information is exchanged  
3 between an error correction decoder and a channel decoder. However, all current  
4 implementations of these soft information exchange schemes use a forward/backward,  
5 Viterbi, or similar decoding algorithm for the decoding function. Specific examples are the  
6 forward/backward and soft output Viterbi algorithm for convolutional codes. In such  
7 decoding schemes, the complexity of the decoder is a design choice, but the complexity of  
8 a trellis based equalizer is a function of the length of the channel impulse response. This  
9 limits practical usefulness of the conventional decoding schemes to a class of channels  
10 having reasonably short impulse response lengths or small signal constellations.

11           The complex nature of conventional equalizing schemes in soft input exchange  
12 methods and devices also places certain physical limitations on devices which rely on such  
13 methods and devices for channel decoding. For example, there exist no solutions beyond one  
14 dimension for forward/backward and Viterbi algorithms. This dictates a one dimensional  
15 data stream, such as the stream obtained by a magnetic disk drive head or an optical disk  
16 drive. There is no significant physical reason for restriction to one-dimensional streams of  
17 data, but an as yet insurmountable hurdle for extension to higher dimensions is the lack of  
18 higher dimensional forward/backward and Viterbi algorithms. A decoding method with  
19 multi-dimensional decoding capability would free devices from the requirement of creating  
20 a one dimensional stream of data. For example, two dimensional regions of disk drives could  
21 be read simultaneously if two dimensional channel decoding is supported. The  
22 forward/backward and Viterbi algorithms used for data decoding are unable to handle the  
23 multidimensional task since multidimensional solutions are unknown.

24           Accordingly, there is a need for an improved method for decoding data  
25 received over a potentially noisy channel which addresses some or all of the aforementioned  
26 drawbacks. It is an object of the invention to provide such an improved method.

## SUMMARY OF THE INVENTION

These and other needs and objects are met or exceeded by the present iterative MMSE equalization-decoder soft information exchange decoding method and device. The method uses a MMSE equalizer which receives and outputs soft information. The equalizer exchanges soft information with a soft information decoder. The simple nature of the equalizer permits solutions beyond one-dimensional data streams and for channels of arbitrary length and for signal constellations of arbitrary size.

## BRIEF DESCRIPTION OF THE DRAWING

Other features, objects and advantages of the invention will be apparent to artisans from the detailed description and the FIGURE, which is a block diagram of a data communication system including an iterative MMSE equalization-decoder for soft information exchange decoding.

## DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENT

The invention is directed to a method for data recovery by confidence building. The invention is applicable, for example, to communications, telephony, recording, data transmission, and restoration. For example, even when the data has not been protected by an error correcting code, there may be other opportunities for confidence building decoders to operate. Suppose that the data were transmitted over multiple channels to the same receiver. This might arise when a preferred user in a cellular telephone system was given two or more cellular phone channels (for example multiple time-slots in a time-division multiple access scheme, multiple frequency slots in a frequency-division multiple access, or multiple spreading codes in a code division multiple access scheme), it might include storing the information on a data recording device multiple times (modular redundancy), or a receiver that has multiple antennas which can receive the same data at multiple locations.

According to the invention, even without error correction coding, a confidence building data recovery system may be constructed, in which equalizers for each of the different channels exchange their beliefs, or estimates, of the data, until a consensus is achieved. A typical and more general case arises when there is one or multiple channels over which data is sent, and the data is protected by an error correction code, likely different for each channel, before being sent over each channel.

The generality of the confidence building data recovery method of the invention will be apparent to artisans. The method will be illustrated with reference to an error correction coded communication channel. An exemplary communication system including such a channel is shown in FIG. 1.

For simplicity, we consider transmission of binary digits  $b_i$  using a Binary Phase Shift Keying (BPSK) modulation scheme. Artisans will understand that the invention is readily generalized to other modulation schemes. We assume that the bits  $b_i$  are encoded in a forward error correction code  $C$  (typically a convolutional code), by an encoder 10, and that the bit stream is permuted by a suitable (e.g. random) interleaver 12 before the transmission.

The encoded symbols are denoted  $\tilde{a}_i$ .  $\Pi$  is a permutation of the bits  $\tilde{a}_i$ . The transmitted signal may be expressed as

$$s(t) = \sum_i b_i h_s(t - iT) \exp(j(2\pi f_0 t + \phi_0))$$

where  $f_0$  is the carrier frequency and  $\phi_0$  is a constant angle. The received signal equals

$$x(t) = \sum_i b_i h_s(t - iT) \exp(j(2\pi f_0 t + \phi_1)) + w(t),$$

where  $\phi_1$  is a constant angle and  $w(t)$  is an additive noise term. In addition to intersymbol interference, a channel 16 typically also adds a noise component. The signal  $x(t)$  is

demodulated, passed through a receiving filter and sampled leading to the equivalent discrete-time model of the communication link which is depicted in FIG. 1.

$$x[u] = \sum_i b_i h[n - i] + w[n],$$

where  $h[j]$  are the sampled values of the overall impulse response of the communication system.

A discrete-time model of the channel 16 with intersymbol interference and additive noise  $w_n$  is described as

$$x[n] = b_n h[0] + w[n] + \sum_{i \neq n} b_i h[n - i].$$

where the term  $\sum_{i \neq n} b_i h[n - i]$  is usually referred to as intersymbol-interference.

We note that the received sampled signal  $x[n]$  is a noisy version the convolution of the bit-sequence with the overall impulse response. In particular, for a finite length impulse response  $h[j]$  the sequence  $x[n]$  can be represented as a noisy output of a Markov process. The task of estimating symbols  $b_i$  in the absence of a coding constraint is referred to as equalization. The invention concerns joint decoding and equalization of the bit-stream  $x[n]$  in an iterative decoder 20 having a soft MMSE equalizer 22 exchanging information with a soft decoder 24.

It is well known how to optimally estimate  $b_n$  from the sequence  $x[n]$  using the so-called forward/backward algorithm. However, the complexity of this approach is usually impractical because the complexity is essentially the product of the complexities of an optimal decoding procedure and an optimal equalization only procedure. The complexity of an optimal equalization only procedure grows exponentially in the length of the overall

1 impulse response, which, for practical channels with impulse response lengths in the tens to  
2 hundreds, is infeasible.

3 Iterative (Turbo) decoding and equalization uses at least two soft input/soft  
4 output (SISO) devices that communicate information. A SISO equalizer takes as input the  
5 received sequence  $x[n]$  and some additional information about the symbol sequence  $b_i$ . This  
6 additional information could for example be a prior probability on the bits. The SISO  
7 equalizer produces a soft estimate of the symbols. This soft output could for example be a  
8 probability mass function indicating the likelihood of any symbol  $b_i$  being equal to 0 or 1.

9 A second SISO block performs the decoding of the estimated bit sequence with  
10 respect to the code. In other words the soft input decoder takes as input reliability  
11 information (typically a probability mass function) and produces as soft output a probability  
12 mass function indicating the likelihood of any symbol  $b_i$  being equal to 0 or 1. The input  
13 reliability information for the equalizer (decoder) is denoted  $\pi_{in}^E(\pi_{in}^D)$ . The output of the  
14 devices is denoted as  $\pi_{out}^E(\pi_{out}^D)$ . For an overall channel impulse response  $h[j]$ , let the  
15 received values  $x[n]$  and reliability vectors  $\pi_{in}[n]$   $n = 1 \dots N$  be given. Without loss of  
16 generality we assume that  $0 \leq \pi_{in}[j] \leq 1$  for all  $j \in \{0 \dots N\}$ . (The  $\pi_{in}[j]$  can be thought of  
17 as prior probabilities).

18 We define a SISO equalizer mathematically as a device that performs an  
19 equalization function  $eq : \mathbb{R}^N \times [0, 1]^N \rightarrow [0, 1]^N$   $eq(y, \pi_{in}^E) \mapsto \pi_{out}^E$ , where  $\pi_{out}^E$  can be thought  
20 of as a vector of a posteriori probabilities.

21 Similarly, we mathematically define an SISO decoder as a device that performs  
22 a decoding function  $dec : [0, 1]^N \rightarrow [0, 1]^N$   $dec(\pi_{in}^D) \mapsto \pi_{out}^D$ , where  $\pi_{out}^D$  can be thought of as  
23 a vector of a posteriori probabilities.

24 The main idea of conventional iterative joint equalization and decoding is to  
25 let the two SISO blocks exchange information until they agree on a decision or until a  
26 maximal number of iterations is reached. As soon as the predetermined termination criterion

is satisfied, the output and the input of the SISO decoder are combined in order to form a soft combined equalization decoding decision. If desired, a hard decision on any symbol can be derived from this soft value.

Typically after a suitable number of iterations the output of the joint equalizer decoder is formed by multiplying the values of the likelihood ratios  $\pi_{in}^D/(1 - \pi_{in}^D)$  and  $\pi_{out}^D/(1 - \pi_{out}^D)$ .

In order to achieve good performance in such an iterative scheme, a permutation is included in the data path, which spreads out statistical dependencies between estimates. The exchange between decoder and equalizer establishes an inherent feedback loop. It is important that this feedback be mitigated as much as possible. We say that an equalizer (decoder) is well-behaved if  $\pi_{out}^E[j]$  is not a function of  $\pi_{in}^E[j]$  ( $\pi_{out}^D[j]$  is not a function of  $\pi_{in}^D[j]$ ).

We now can describe the iterative equalization/decoding scheme in the following algorithm:

1. INPUT: A permutation  $\Pi$ , a vector of received values  $y$ , and prior information about the symbols expressed as a vector of probabilities  $\pi_{in}^E$  ( $\pi_{in}^E$  is usually a vector containing a value 0.5 in each position.)

2. Repeat the following steps until a termination criterion is reached.

(a)  $\pi_{out}^E = eq(y, \pi_{in}^E)$

(b)  $\pi_{in}^D = \Pi^{-1}(\pi_{out}^E)$

(c)  $\pi_{out}^D = dec(\pi_{in}^D)$

(d)  $\pi_{in}^E = \Pi(\pi_{out}^D)$

(e) If the termination criterion is not satisfied go to step 2a.

3. OUTPUT hard decisions for symbol  $b_i$  comparing  $\frac{\pi_{out}^D[i]\pi_{in}^D[i]}{(1 - \pi_{out}^D[i])(1 - \pi_{in}^D[i])}$  to 1.



Iterative joint equalization and decoding schemes can be compared on the basis of the above algorithm and various combinations of decoding functions and equalization functions have been reported on. As decoding functions we explicitly mention the forward-backward and the SOVA algorithm for convolutional codes. Adaptations of these algorithms have been employed in the equalizer function. However, while the complexity of the decoder is a design issue, the complexity of a trellis based equalizer is based on the length of the channel impulse response. This severely hampers practical use of iterative equalization decoding techniques based on the turbo principle. For channels with long impulse responses, the exclusively used equalization techniques are either linear equalization or decision feedback techniques without use of the decoder in the equalization process. The invention solves this problem by providing an equalization algorithm independent of the length of the channel impulse response that can be used in iterative decoding. In particular, a minimum mean square error equalizer is demonstrated by the invention to facilitate joint decoding and equalization. The iterative technique of the invention is applicable to channels with a short or long impulse response, and with small or large signal constellations.

Let  $b[n]$  be a sequence of symbols that have been transmitted through a communications channel, such as a wireless or wireline data link, or recorded onto some data recording medium, such as an optical ROM or magnetic disk or tape. A linear, equivalent base-band model for the received (or read-back) sequence of symbols is given by

$$x[n] = \sum_{k=-L_1}^{L_2} h[k]b[n-k] + w[n],$$

where  $h[k]$  is the length  $L_1 + L_2 + 1$  impulse response of the channel, and  $w[n]$  is an additive noise term modeling electrical and thermal fluctuation and unmodeled components of the channel. This generic linear model is a widely accepted mathematical model for the

distortions induced on a sequence of symbols both in communications and data storage applications. A linear equalizer for this channel can be used to attempt to reduce the effects of the intersymbol interference and the additive noise induced by the channel. A linear (affine) equalizer with coefficients  $c[n, k]$  and offset  $g[n]$  and symbol estimate  $\hat{b}[n]$  can be expressed in the form

$$\hat{b}[n] = \sum_{k=-N_1}^{N_2} c[n, k]x[n+k] + g[n],$$

where the equalizer coefficients are written as a function  $n$  to enable the possibility of different coefficients used to estimate each symbol  $\hat{b}[n]$ , and the offset  $g[n]$  provides a richer class of linear estimates which can account for a non-zero mean prior. The channel model can be written in matrix form as

$$\vec{x}[n] = H\vec{b}[n] + \vec{w}[n],$$

where the channel response matrix,  $H$ , is given by

$$\begin{bmatrix} h[L_2] & h[L_2-1] & \dots & h[-L_1] & 0 & \dots & 0 \\ 0 & h[L_2] & h[L_2-1] & \dots & h[-L_1] & \dots & 0 \\ \vdots & & & & & & \\ 0 & \dots & 0 & h[L_2] & h[L_2-1] & \dots & h[-L_1] \end{bmatrix},$$

and the signal vectors are given by

$$\vec{x}[n] = [x[n - N_1] \dots x[n] \dots x[n + N_2]]^T,$$

$$\vec{b}[n] = [b[n - N_1 - L_2] \dots b[n] \dots b[n + N_2 + L_1]]^T,$$

$$\vec{w}[n] = [w[n - N_1] \dots w[n] \dots w[n + N_2]]^T.$$

The equalizer output can also be expressed in matrix form, simply as

$$\hat{b}[n] = \vec{c}[n]^T \vec{x}[n] + g[n],$$

where the equalizer coefficients  $c[n,k]$  are written

$$\vec{c}[n] = [c[n, -N_1], \dots, c[n, N_2]].$$

With this channel model, the mean-square error of a symbol estimate  $b[n]$  based on the estimate  $\hat{b}[n]$  is given by

$$E\{|b[n] - \hat{b}[n]|^2\}, \quad (2)$$

where the expectation is taken over the distribution of the symbols  $b[n]$  and the noise  $w[n]$ . In the conventional traditional approach to the design of complexity-constrained (finite  $N_1$  and  $N_2$ ) minimum mean-square error (MMSE) linear equalizers, it is assumed that the symbols  $b[n]$  are equally likely to take on all possible symbol values, and that there is no additional information about their values available. The conventional equalizer is determined by finding the coefficient values  $\vec{c}[n]$  and  $g[n]$  which minimize the mean squared error (2), which, since the symbols are assumed unknown and equally likely for all time  $n$  leads to a single set of coefficients,  $\vec{c}$ , and the offset is given by  $g[n] = 0$ .

According to the invention, the equalizer 22 implements an equalization algorithm which has available a set of priors over the symbols. For example, if the symbol alphabet is binary, then this would correspond to the availability of the sequence  $\pi_{in}^E[n] = \text{Prob}\{b[n]=1\}$ . In the sequel, we assume that the channel response  $h[n]$  is real and that the symbol alphabet is  $b[n] \in \{-1, 1\}$  for simplicity. Extension to complex baseband channels and higher-order symbol constellations is straightforward. In this case, the MMSE equalizer 22 can be designed incorporating these priors into the optimization. Hence, the equalizer coefficients,  $\vec{c}[n]$  and  $g[n]$  can be determined by finding the minimum of the mean-squared error (2), where the expectation in (2) is over both the additive noise in the channel, and the

given (time-varying) prior over the symbols. As a result, the equalizer coefficients will vary with time index,  $n$ . This leads to the following formulation,

$$\hat{b}[n] = E\{b[n]\} + [E\{b[n]\tilde{b}[n]\}H^T - E\{b[n]\}E\{\tilde{b}[n]^T\}H^T] \\ [HE\{\tilde{b}[n]\tilde{b}[n]^T\}H^T + E\{|w[n]|^2\}I - HE\{\tilde{b}[n]\}E\{\tilde{b}[n]^T\}H^T]^{-1} (\bar{x}[n] - HE\{\tilde{b}[n]\}).$$

Once the equalizer has produced MMSE linear estimates of the symbols  $\hat{b}[n]$ , these estimates must be mapped into priors  $\pi_{out}^E$ . One method for mapping the outputs of the linear equalizers is to assume the output distribution  $\hat{b}[n]$  is conditionally Gaussian, distributed about the symbol values. This leads to the following mapping

$$Prob\{b[n] = 1 | \hat{b}[n]\} = \frac{1}{2} \left( 1 + \tanh \left( \frac{\hat{b}[n]}{\sigma_b^2} \right) \right),$$

where  $\sigma_b^2$  is the variance of the conditional output distribution given the symbol  $\hat{b}[n] = \text{sign}(\hat{b}[n])$ . In order for this equalizer to be considered well-behaved, the estimate  $\hat{b}[n]$  cannot be a function of  $\pi_{in}^E[n]$ . Hence, the expectations must be taken over a distribution of the symbols which excludes  $\pi_{in}^E[n]$  for the calculation of  $\hat{b}[n]$ . However, in calculating  $\hat{b}[k]$ ,  $k \neq n$ ,  $\pi_{in}^E[n]$  may be used. This leads to the following method for computing the output distribution given the observations,  $x[n]$  and the input distribution  $\pi_{IN}^E$ .

## CREATE BUFFERS

1. Create buffers for the priors, the signal  $x[n]$ , the expectations  $\vec{bb}[n] = E\{b[n]\tilde{b}[n]\}$ , the correlation matrix  $B[n] = E\{\tilde{b}[n]\tilde{b}[n]^T\}$ , and the means  $\vec{mb}[n] = E\{\tilde{b}[n]\}$
- $$\vec{\pi}^{(n)} \triangleq [\pi^{(n)}[-N_1 - L_2], \dots, \pi^{(n)}[0], \dots, \pi^{(n)}[N_2 + L_1]]^T$$
- $$\vec{x}^{(n)} \triangleq [x^{(n)}[-N_1], \dots, x^{(n)}[0], \dots, x^{(n)}[N_2]]^T$$
- $$\vec{bb}^{(n)} \triangleq [bb^{(n)}[-N_1 - L_2], \dots, bb^{(n)}[0], \dots, bb^{(n)}[N_2]]^T = [0, \dots, 0, 1, 0, \dots, 0]^T$$

2. Initialize buffers for priors  $\vec{\pi}^{(n)}$  and data  $\vec{x}^{(n)}$ , in terms of the signal  $x[n]$  and the input  $\pi_{IN}^E$ .

$$\vec{x}^{(0)} = [0, 0, \dots, x[0], x[1], \dots, x[N_2]]^T$$

$$\vec{\pi}^{(0)} = [0, 0, \dots, 0, \pi_{IN}^E[0], \pi_{IN}^E[1], \dots, \pi_{IN}^E[N_2 + L_1]]^T$$

3. Loop over the data for  $n = 0, \dots, N$ :

$$\pi^{(n)}[0] = 1/2$$

$$\vec{mb}^{(n)} = 2 \vec{\pi}^{(n)} - 1$$

$$B = \vec{mb}^{(n)} \vec{mb}^{(n)T}$$

$$\text{diag}(B) = \text{diag}(1, 1, \dots, 1)$$

$$\vec{c}[n] = [H (B - \vec{mb}^{(n)} \vec{mb}^{(n)T}) H^T + \sigma_w^2 I]^{-1} H \vec{bb}^{(n)}$$

$$\hat{b}[n] = \vec{mb}^{(n)} + \vec{c}^{(n)T} (\vec{x}^{(n)} - H \vec{mb}^{(n)})$$

$$\vec{x}^{(n+1)} = [x^{(n)}[-N_1 + 1], \dots, x^{(n)}[N_2], 0]$$

$$\vec{\pi}^{(n+1)} = [\pi^{(n)}[-N_1 - L_2 + 1], \dots, \pi^{(n)}[N_2 + L_1], 0]$$

if  $n < N - N_2$

$$x^{(n+1)}[N_2] = x[n + 1 + N_2]$$

if  $n < N - N_2 - L_1$

$$\pi^{(n+1)}[N_2 + L_1] = \pi_{IN}^E[n + 1 + N_2 + L_1]$$

4. Estimate output variance  $\sigma_b^2 = (\text{var}(\hat{b} | \hat{b} > 0) + \text{var}(\hat{b} | \hat{b} < 0))/2$

5. Determine output priors,  $\pi_{OUT}^E = 1/2 (1 + \tanh(\frac{\hat{b}[n]}{\sigma_b^2}))$

## SOFT-INPUT SOFT-OUTPUT DECISION FEEDBACK EQUALIZATION

For a minimum mean-square error (MMSE) decision feedback equalizer, the channel model can be written in similar matrix form to the MMSE linear equalizer,

$$\begin{bmatrix} \vec{x}[n] \\ \vec{d}[n] \end{bmatrix} = \begin{bmatrix} H \\ I_{M \times M} | 0_{M \times M} \end{bmatrix} \vec{b}[n] + \begin{bmatrix} \vec{w}[n] \\ 0_{M \times 1} \end{bmatrix},$$

where, for simplicity of notation, it is assumed that the number of decision symbols feedback to the equalizer is given by  $M = N_1 + L_2$ , for  $\vec{d}[n] = [d[n-1], \dots, d[n-M]]$ . For binary antipodal signaling,  $d[n-k] = \text{sign}(\hat{b}[n-k])$ , and for higher order signaling constellations, a suitable quantizer to the nearest symbol would be used.

The MMSE DFE coefficients are then given by

$$\vec{b}[n] = E\{b[n]\} + \left[ E\{b[n]\vec{b}[n]\}H^T - E\{b[n]E\{\vec{b}[n]^T\}H^T \mid 0_{1 \times M} \right]$$

$$\left[ \frac{HE\{\vec{b}[n]\vec{b}[n]^T\}H^T + E\{w[n]^2\}I - HE\{\vec{b}[n]\}E\{\vec{b}[n]^T\}H^T HE\{\vec{b}[n]\vec{d}[n]^T\} - HE\{\vec{b}[n]\}E\{\vec{d}[n]^T\}}{E\{\vec{d}[n]\vec{b}[n]^T\}H^T + E\{\vec{d}[n]\}E\{\vec{b}[n]^T\}H^T E\{\vec{d}[n]\vec{d}[n]^T\} - E\{\vec{d}[n]\}E\{\vec{d}[n]^T\}} \right]^{-1}$$

$$\begin{bmatrix} \vec{x}[n] - HE\{\vec{b}[n]\} \\ \vec{d}[n] - E\{\vec{d}[n]\} \end{bmatrix}$$

When this MMSE DFE is made well-behaved, we set  $E\{b[n]\} = 0$ , which, together with some algebra, reduces this expression considerably, to

$$\hat{b}[n] = E\{b[n]\vec{b}[n]^T\}H^T \left[ HE\{\vec{b}[n]\vec{b}[n]^T\}H^T + E\{w[n]^2\}I - HE\{\vec{b}[n]\}E\{\vec{b}[n]^T\}H^T - \right.$$

$$\left. (HE\{\vec{b}[n]\vec{d}[n]^T\} - HE\{\vec{b}[n]\}E\{\vec{d}[n]^T\})E\{\vec{d}[n]\vec{d}[n]^T\}^{-1}(E\{\vec{d}[n]\vec{b}[n]^T\}H^T - E\{\vec{d}[n]\}E\{\vec{b}[n]^T\}H^T) \right]^{-1}$$

$$\left[ (\vec{x}[n] - HE\{\vec{b}[n]\}) - (HE\{\vec{b}[n]\vec{d}[n]^T\} - HE\{\vec{b}[n]\}E\{\vec{d}[n]^T\})E\{\vec{d}[n]\vec{d}[n]^T\}^{-1}(\vec{d}[n] - E\{\vec{d}[n]\}) \right].$$

1           The preferred implementation of the present invention is shown in the  
2 FIGURE. In the FIGURE, a process for encoding/transmission and reception/decoding  
3 begins with a set of digital data, depicted as data bits  $a_i$ . These data bits are then encoded,  
4 using forward error correction coding 10, to produce the encoded sequence of bits, depicted  
5 as  $\tilde{a}_i$ .

6           The encoded data bits are now interleaved (re-ordered) in time using the data  
7 interleaver 12. The purpose of the data interleaver is to re-order the data such that the  
8 statistical dependencies between the data bits are spread out in time. This makes adjacent  
9 data bits in the re-ordered sequence,  $b_i$  in the figure, approximately independent of one  
10 another. The re-ordered bits  $b_i$  are now ready to be transmitted over the channel 16. The  
11 process of mapping the bits  $b_i$  into channel symbols and transmitting them over the channel  
12 is depicted in the FIGURE as block 16. The ISI channel 16 introduces distortion into the  
13 sequence of channel symbols. The channel 16 also is assumed to exhibit additive noise, as  
14 depicted in the FIGURE. The output of the channel 16, is then the sequence of corrupted  
15 channel symbols,  $x[n]$ . The received sequence  $x[n]$  is then processed by the receiver block  
16 20 to remove effects of the channel. The receiver block comprises several elements.

17           First, the received data symbols are equalized using a soft-input/soft-output  
18 MMSE equalizer 22. The equalizer 22 attempts to eliminate the intersymbol interference  
19 (ISI) from the channel. The output of the equalizer 22 is a set of priors, or confidence levels,  
20 in the symbol values, labeled as  $\Pi_{OUT}^E$  in the figure. The two inputs to the SISO MMSE  
21 equalizer 22 are the channel output symbols  $x[n]$  and a set of confidence levels in their  
22 values,  $\Pi_{IN}^E$ . On the first pass through the equalizer, an initialization set of confidence levels  
23 are used, which are equally-likely to take on all values. Subsequent passes use confidence  
24 levels produced by the SISO decoder 24.

25           The confidence levels are then re-ordered using the de-interleaver 12a to place  
26 them in the same order as the corresponding bits in the encoded sequence  $a_i$ . The confidence

1 levels can now be used in a soft-input/soft-output decoder 24 to produce estimates of the  
2 original uncoded data bits  $a_i$ , labeled as  $II_{OUT}^E$  in the figure. The confidence levels over the  
3 sequence  $a_i$  are then interleaved 12 again, back to the ordering of the channel symbols and  
4 the interleaved data bits  $b_i$ . The confidence levels are now used as input to the SISO MMSE  
5 equalizer 22, together with the data  $x[n]$ . Whereas in the first pass through the equalizer 22,  
6 the confidence levels were arbitrarily preset to initialized values, now the confidence levels  
7 have been determined by the decoding process. This cycle is repeated until either a  
8 convergence criteria is met, or a sufficient number of passes over the data have elapsed.  
9 Typically, the convergence criteria will consist of a prespecified measure of match between  
10 the confidence levels determined by the decoding process and those determined by the  
11 equalization process. Other possible convergence criteria could include testing that the  
12 confidence levels determined by one or the two SISO devices have not changed appreciably  
13 over a sequence of passes.

14 While various embodiments of the present invention have been shown and  
15 described, it should be understood that other modifications, substitutions and alternatives are  
16 apparent to one of ordinary skill in the art. Such modifications, substitutions and alternatives  
17 can be made without departing from the spirit and scope of the invention, which should be  
18 determined from the appended claims.

19 Various features of the invention are set forth in the appended claims.



WHAT IS CLAIMED IS:

1           1.     A method for estimating digital data received over a potentially noisy  
2 channel which adds intersymbol interference or additive noise, or a combination of  
3 intersymbol interference or additive noise, the method comprising steps of:

4           inputting data received from the noisy channel into a SISO MMSE equalizer;  
5           inputting a set of priors over symbol values of the noisy channel including a  
6 separate prior for each received noisy channel symbol value, into the SISO MMSE equalizer;  
7           equalizing, by an MMSE equalization in the SISO MMSE equalizer, the data  
8 received from the noisy channel and the set of priors over symbol values to produce a symbol  
9 value estimate;

10           mapping output of the SISO MMSE equalizer onto priors over the symbol  
11 values to produce a confidence indication in each of the symbol value estimates as a function  
12 of time.

1           2.     The method according to claim 1, further comprising a step of setting  
2 parameters of the SISO MMSE equalizer according to MMSE criterion over statistics of the  
3 channel noise and statistics of the symbol values.

1           3.     The method according to claim 1, wherein digital data transmitted over  
2 said potentially noisy channel is error correction encoded prior to transmission, the step of  
3 mapping output of the SISO MMSE equalizer comprising steps of:

4           passing output of the SISO MMSE equalizer into a SISO error correction  
5 decoder;

6           using an output of said SISO error correction decoder as the set of priors over  
7 symbol values;

8 repeating all steps of the method until a predetermined convergence criterion  
9 is reached between said SISO error correction decoder and said SISO MMSE equalizer.

1 4. The method according to claim 3, wherein digital data transmitted over  
2 said potentially noisy channel is interleaved prior to transmission and a step of de-  
3 interleaving is conducted on the output of the SISO MMSE equalizer prior to said step of  
4 passing output of the SISO error correction decoder is interleaved prior to said step of  
5 repeating.

1 5. The method according to claim 4, wherein said step of equalizing  
2 excludes symbol value estimates which are functions of an input distribution of a current  
3 symbol being equalized.

1 6. The method according to claim 5, wherein said SISO error correction  
2 decoder has its output restricted to exclude symbol value estimates which are functions of  
3 an input distribution of a current symbol being decoded.

1 7. The method according to claim 1, wherein said step of equalizing  
2 comprises a fast update equalization of the order  $M^2$  (quadratic in the number of parameters  
3  $M$ ) which exploits redundant computations in successive equalizer computations.

1 8. The method according to claim 7, wherein the fast update equalization  
2 is performed by applying the matrix inversion lemma to a matrix to be inverted in a design  
3 of equalization coefficients for the SISO MMSE equalizer.

1           9.     The method according to claim 1, wherein said data is multi-  
2 dimensional and is error correction encoded then interleaved prior to transmission in the  
3 channel, and wherein said SISO MMSE equalizer handles single-input multiple output data,  
4 the mapping step comprising steps of:

5                 de-interleaving outputs of the SISO MMSE equalizer and re-serializing data  
6 output from the SISO MMSE equalizer into one-dimensional encoding;

7                 SISO decoding the re-serialized output of the SISO MMSE equalizer; and

8                 repeating said equalizing and mapping steps until a predetermined convergence  
9 criterion is met between said equalizing and SISO decoding steps.

10           10.    A method for equalization and decoding of digital data received over  
11 a multiple noisy channels which each add intersymbol interference or additive noise, or a  
12 combination of intersymbol interference or additive noise, wherein data transmitted over each  
13 channel is interleaved prior to transmission, the method comprising steps of:

14                 performing a soft equalization for each channel by

15                 inputting data received from said one of the noisy channels into a SISO MMSE  
equalizer;

                  inputting a set of priors over symbol values of the noisy channel including a  
separate prior for each received noisy channel symbol value, into the SISO MMSE equalizer;

                  equalizing, by an MMSE equalization in the SISO MMSE equalizer, the data  
received from the noisy channel and the set of priors over symbol values; and

                  mapping output of the SISO MMSE equalizer onto priors over the symbol  
values to produce a confidence indication in each of the symbol values as a function of time,

                  then, using de-interleaved output from said mapping step for one channel, iteratively  
decoding information for a second channel by repeating said performing a soft equalization

while substituting said de-interleaved output for said set of priors and substituting output from said mapping step for said second channel for said data received until a predetermined convergence criterion is reached.

11. A data decoding device comprising:  
 a SISO MMSE equalizer;  
 a SISO decoder, the decoder exchanging symbol estimates with the SISO MMSE equalizer, the SISO MMSE equalizer produces MMSE linear estimates of transmitted symbols  $\hat{b}[n]$ , and mapping the linear estimates to an output set of priors over the symbols  $\pi_{OUT}^E$ .

12. The device according to claim 11, wherein said equalizer maps the estimates by treating the output distribution  $\hat{b}[n]$  as conditionally Gaussian, and distributed about the symbol values.

13. The device according to claim 12, wherein the output distribution mapping is defined as:

$$Prob\{b[n] = 1 | \hat{b}[n]\} = \frac{1}{2} \left( 1 + \tanh \left( \frac{\hat{b}[n]}{\sigma_b^2} \right) \right),$$

where  $\sigma_b^2$  is the variance of the conditional output distribution given the symbol  $\hat{b}[n] = \text{sign}(\hat{b}[n])$ . and the estimate  $\hat{b}[n]$  cannot be a function of  $\pi_{IN}^E[n]$ , and expectations are taken over a distribution of the symbols which excludes  $\pi_{IN}^E[n]$  for the calculation of  $\hat{b}[n]$ .

14. The device according to claim 12, wherein the following steps for computing the output distribution given the observations,  $x[n]$  and the input distribution  $\pi_{IN}^E$  is used by the MMSE equalizer:

a. Create buffers for the priors, the signal  $x[n]$ , the expectations  $\vec{bb}[n] = E\{b[n]\bar{b}[n]\}$ , the correlation matrix  $B[n] = E\{\bar{b}[n]\bar{b}[n]^T\}$ , and the means  $\vec{mb}[n] = E\{\bar{b}[n]\}$

$$\vec{\pi}^{(n)} \triangleq [\pi^{(n)}[-N_1 - L_2], \dots, \pi^{(n)}[0], \dots, \pi^{(n)}[N_2 + L_1]]^T$$

$$\vec{x}^{(n)} \triangleq [x^{(n)}[-N_1], \dots, x^{(n)}[0], \dots, x^{(n)}[N_2]]^T$$

$$\vec{bb}^{(n)} \triangleq [bb^{(n)}[-N_1 - L_2], \dots, bb^{(n)}[0], \dots, bb^{(n)}[N_2]]^T = [0, \dots, 0, 1, 0, \dots, 0]^T$$

b. Initialize buffers for priors  $\vec{\pi}^{(n)}$  and data  $\vec{x}^{(n)}$ , in terms of the signal  $x[n]$  and the input  $\pi_{IN}^E$ .

$$\vec{x}^{(0)} = [0, 0, \dots, x[0], x[1], \dots, x[N_2]]^T$$

$$\vec{\pi}^{(0)} = [0, 0, \dots, 0, \pi_{IN}^E[0], \pi_{IN}^E[1], \dots, \pi_{IN}^E[N_2 + L_1]]^T$$

c. Loop over the data for  $n=0, \dots, N$ :

$$\pi^{(n)}[0] = 1/2$$

$$\vec{mb}^{(n)} = 2 \vec{\pi}^{(n)} - 1$$

$$B = \vec{mb}^{(n)} \vec{mb}^{(n)T}$$

$$\text{diag}(B) = \text{diag}(1, 1, \dots, 1)$$

$$\vec{c}[n] = [H (B - \vec{mb}^{(n)} \vec{mb}^{(n)T}) H^T + \sigma_w^2 I]^{-1} H \vec{bb}^{(n)}$$

$$\hat{b}[n] = \vec{mb}^{(n)} + \vec{c}^{(n)T} (\vec{x}^{(n)} - H \vec{mb}^{(n)})$$

$$\vec{x}^{(n+1)} = [x^{(n)}[-N_1 + 1], \dots, x^{(n)}[N_2], 0]$$

$$\vec{\pi}^{(n+1)} = [\pi^{(n)}[-N_1 - L_2 + 1], \dots, \pi^{(n)}[N_2 + L_1], 0]$$

$$\text{if } n < N - N_2$$

$$x^{(n+1)}[N_2] = x[n + 1 + N_2]$$

$$\text{if } n < N - N_2 - L_1$$

$$\pi^{(n+1)}[N_2 + L_1] = \pi_{IN}^E[n + 1 + N_2 + L_1]$$

d. Estimate output variance  $\sigma_b^2 = (\text{var}(\hat{b} | \hat{b} > 0) + \text{var}(\hat{b} | \hat{b} < 0))/2$

- e. Determine output priors,  $\pi_{OUT}^E = 1/2 (1 + \tanh(\frac{\hat{b}[n]}{\sigma_{\hat{b}}^2}))$ .

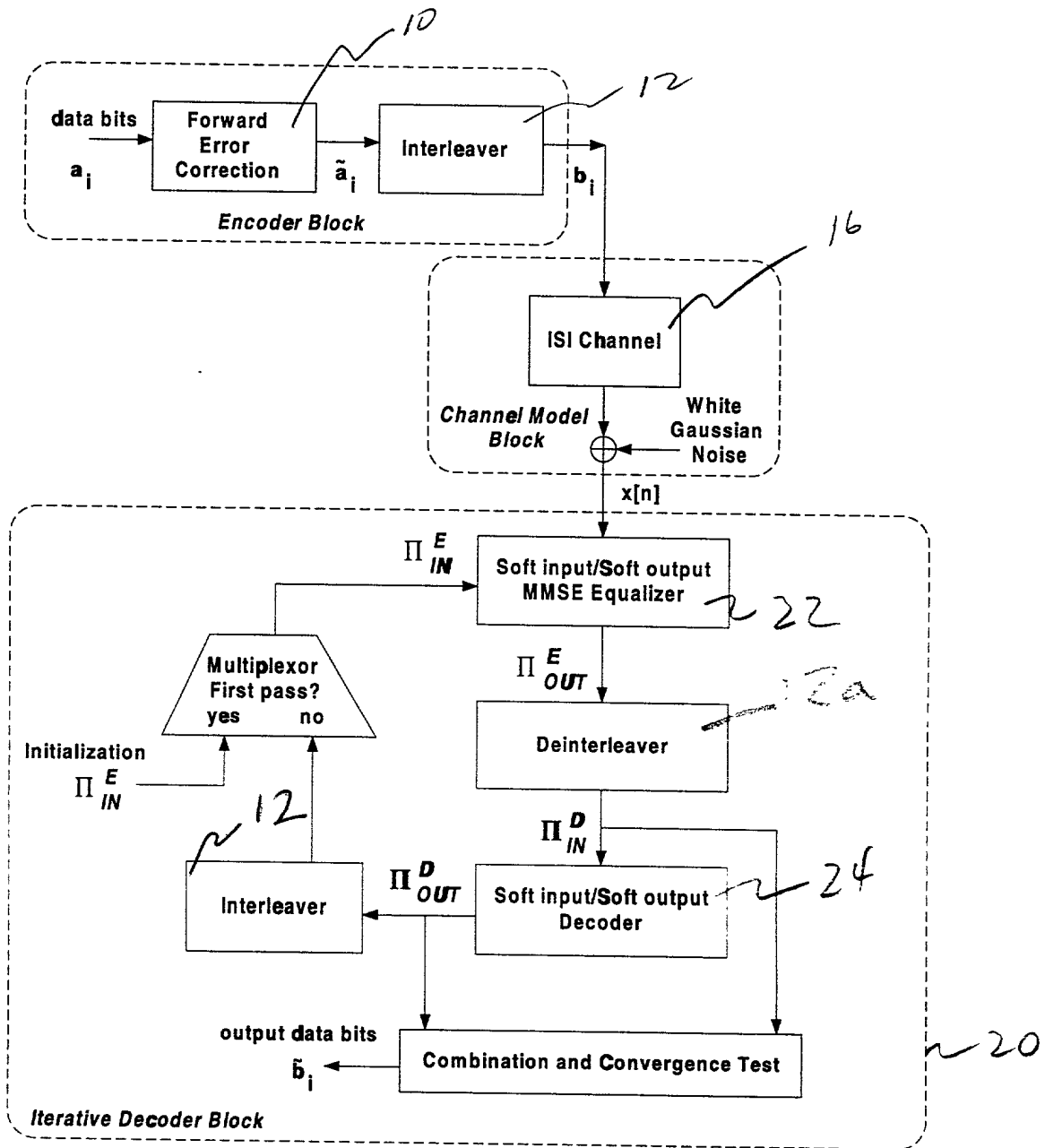
1                    ITERATIVE MMSE EQUALIZATION-DECODER SOFT  
2                    INFORMATION EXCHANGE DECODING METHOD AND DEVICE

3                    ABSTRACT OF THE DISCLOSURE

4                    A method and device employing an iterative MMSE equalization-decoder soft  
5                    information exchange decoding. The method uses a MMSE equalizer which receives and  
6                    outputs soft information. The equalizer exchanges soft information with a soft input soft  
7                    output decoder, preferably an error correction decoder. The nature of the equalizer permits  
8                    solutions beyond one-dimensional data streams and permits solutions for long channel  
9                    lengths and multi-dimensional data since the solution is not a function of the channel impulse  
10                    response.

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# FIGURE



009T50-40204960



## DECLARATION AND POWER OF ATTORNEY

As a below named inventor, I hereby declare:

That my residence, post office address and citizenship are as stated below next to my name.

That I verily believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural inventors are named below) of the subject matter which is claimed and for which a patent is sought on the invention entitled:

### **ITERATIVE MMSE EQUALIZATION-DECODER SOFT INFORMATION EXCHANGE DECODING METHOD AND DEVICE**

the specification of which (check one)

( X ) is attached hereto.

( ) was filed on \_\_\_\_\_ as  
Application Serial No. \_\_\_\_\_  
and was amended on \_\_\_\_\_  
(if applicable)

That I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above.

That I acknowledge the duty to disclose information known to be material to patentability of this application in accordance with Title 37, Code of Federal Regulations, §1.56(a).

That I hereby claim foreign priority benefits under Title 35, United States Code, §119 of any foreign application(s) for patent or inventor's certificate listed below and have also identified below any foreign application for patent or inventor's certificate on this invention having a filing date before that of the application on which priority is claimed:

#### **Prior Foreign Application(s)**

#### **Priority Claimed**

_____ (Number)	_____ (Country)	_____ (Day/Month/Year Filed)	<input type="checkbox"/> Yes	<input type="checkbox"/> No
_____ (Number)	_____ (Country)	_____ (Day/Month/Year Filed)	<input type="checkbox"/> Yes	<input type="checkbox"/> No
_____ (Number)	_____ (Country)	_____ (Day/Month/Year Filed)	<input type="checkbox"/> Yes	<input type="checkbox"/> No

That I hereby claim the benefit under Title 35, United States Code, §120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, §112, I acknowledge the duty to disclose material information as defined in Title 37, Code of Federal Regulations, §1.56(a) which occurred between the filing date of the prior application and the national or PCT international filing date of this application:

#### **United States Application(s)**

_____ (Application Serial No.)	_____ (Filing Date)	_____ (Status)-(Patented, pending, abandoned)
_____ (Application Serial No.)	_____ (Filing Date)	_____ (Status)-(Patented, pending, abandoned)
_____ (Application Serial No.)	_____ (Filing Date)	_____ (Status)-(Patented, pending, abandoned)

That all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issuing thereon.

I hereby appoint the following attorneys and agents, with full power of substitution and revocation, to prosecute this application and to transact all business in the United States Patent and Trademark Office connected therewith and request that all correspondence and telephone calls in respect to this application be directed to GREER, BURNS & CRAIN, LTD., Suite 8660 - Sears Tower, 233 South Wacker Drive, Chicago, Illinois 60606, Telephone No. (312) 993-0080:

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